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A MATHEMATICAL MODEL OF AIR TEMPERATURE IN MAMMOTH CAVE, KENTUCKY

JONATHAN W. JERNIGAN
Department of Mathematics, Western Kentucky University, Bowling Green, KY 42101, USA Jonathan.Jernigan@wku.edu
RANDALL J. SWIFT
Department of Mathematics, Western Kentucky University, Bowling Green, KY 42101, USA

Alterations made to the Natural (Historic) Entrance into Mammoth Cave over the past two centuries have resulted in disrupted atmospheric conditions in the Historic Section of Mammoth Cave. In an effort to understand atmospheric phenomena in this section of the cave, Division of Science and Resources Management personnel at Mammoth Cave National Park collected atmospheric data from various sites throughout the Historic Section of Mammoth Cave.

These data are used to construct a mathematical model that predicts air temperature at various sites within the cave system. First, an approximate mathematical model is constructed that could apply to any cave system with characteristics (such as cave geometry and the natural force driving airflow) similar to those in Mammoth Cave. Then, the regression analysis of atmospheric data and the use of the derived model allow the construction of a mathematical model that is specific to the Historic Section of Mammoth Cave.

Around 175 years ago, a sequence of structural changes to the Historic Section of Mammoth Cave, Mammoth Cave National Park, Kentucky, was initiated, the effects of which may still be seen today. The primary and most dramatic of these modifications was the clearance of large rockfall debris from the entrance passage known as Houchins Narrows. With a clearer entrance passage, air exchange between the Historic Section of the cave and the outdoors could readily occur. Historically, this air movement had been retarded by entrance structures that restricted airflow into the Historic Section of the cave; air movement, however, was not the only activity that these structures restricted. The movement of bats was also impeded (Olson 1995).

Historical and paleontological data indicate that the Historic Section of Mammoth Cave was once a major bat hibernation site. Several 19th century estimates reported that hibernating colonies totaled into the millions of bats (Olson 1996). Paleontological evidence includes bat bones and excrement found throughout the Historic Section of the cave (Toomey *et al.* 1998). Today, however, few bats are found in this same portion of the cave where they were once so abundant. In an effort to restore the bat hibernacula and atmospheric conditions, the existing gate structure was replaced in July 1990 with a bat gate. The removal of the preexisting structure opened the Historic Section of the cave to the effects of outdoor atmospheric conditions, and the atmospheric conditions within this portion of the cave fluctuated erratically (Olson 1995). The Park's Division of Science and Resources Management has attempted to control this behavior by partially covering the bat gates with Plexiglas panels (Fry 1996).

In order to understand atmospheric phenomena and quantify the effects of these entrance modifications, it is necessary to monitor atmospheric conditions within the Historic Section of Mammoth Cave. The Cave Atmospheric Monitoring program, initiated in 1994, is maintained by the Science and Resources Management division of the National Park Service at Mammoth Cave. The program began not in the Historic Section of Mammoth Cave, but in the ecotone of various artificial entrances to the Mammoth Cave system. The original goal of the monitoring program was to study the effects that airlock installations would have upon atmospheric conditions near each of these artificial entrances. The atmospheric monitoring stations installed a few months later in the Historic Section of Mammoth Cave were placed there in an effort to understand the ecotone in that area by providing baseline data that described atmospheric conditions in that section of the cave (Fry 1995).

The mathematical model created in this paper uses data from eight stations in the Historic Section of Mammoth Cave (Fig. 1). All of the Cave Atmospheric Monitoring (CAM) stations in the Historic Section of Mammoth Cave record air temperature data on fifteen-minute intervals.

The monitoring site that is central to this study is the Houchins Narrows site. Located 103 m from the Natural Entrance, the Houchins Narrows CAM station is the station closest to the Natural Entrance and the station where atmospheric conditions fluctuate most in response to weather patterns external to the cave system. Atmospheric parameters collected at this site include not only air temperature, but wind speed and direction measurements as well. Air flux, or the volume of air flowing past the station per unit time along with a directional indicator, is calculated using these wind speed and direction measurements. Denoted by $\mathbf{F}_H$ [m^3/s], air flux is assigned positive values when air is flowing into the cave and negative values when air is flowing out of the cave. Further, the cross-sectional area of the cave passage is 19.2 m^2 at the

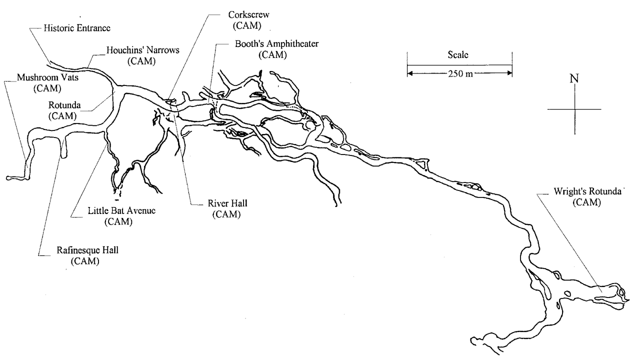

Figure 1. Cave atmospheric monitoring (CAM) stations in the Historic Section of Mammoth Cave (modified after Kaemper 1908).

Houchins Narrows CAM station so that air flux is given by

$$F_H = \begin{cases} 19.2\,m^2 \times \text{Wind Speed} & \text{If Incast} \\ -19.2\,m^2 \times \text{Wind Speed} & \text{If Outcast} \end{cases} \qquad (1)$$

The Natural Entrance is among the largest apertures that lead into the Historic Section of Mammoth Cave. Any disturbance to atmospheric conditions in the Historic Section of Mammoth Cave occurs in response to variations in atmospheric conditions outside the cave system. These variations are registered at the Houchins Narrows CAM station. It can be assumed that atmospheric conditions in Houchins Narrows are closely related to atmospheric conditions at other sites throughout the Historic Section's variable temperature zone. For this reason, the mathematical model derived in this paper uses data from the Houchins Narrows CAM site to predict air temperature data at sites that are deeper inside the Historic Section of Mammoth Cave but still within the variable temperature zone.

To formulate the desired mathematical model, the natural phenomenon that drives airflow in Mammoth Cave must be identified and understood. Empirical evidence indicates that airflow in Mammoth Cave is driven by the chimney effect (Jernigan 1997). The chimney effect is the natural *convection* process whereby air movement occurs due to density differentials between two neighboring air masses of differing temperatures (Wefer 1994). The baseline mathematical model derived in the following section considers a model that relates atmospheric conditions between two sites in the cave system.

A BASELINE MODEL OF AIRFLOW IN MAMMOTH CAVE

As a first step in the mathematical modeling of a complex system, a model is derived that describes the behavior of the system in its simplest form. The physical model about to be presented cannot be applied directly to airflow in the Mammoth Cave system since many of the assumptions underlying the model are violated. However, the model is created not to be applied directly to airflow in Mammoth Cave, but to imply the set of basis functions that is used in the regression analysis of atmospheric data. Since the physical model is applied very loosely to airflow in the Mammoth Cave system, it is not rigorously constructed.

A cave system may be abstracted as a network of interconnecting pipes. So, when considering airflow in the Mammoth Cave system, the mathematical model may be one that depends upon the representation of the cave system as such a system of networked piping. To derive a model of airflow in this branching system, an undivided section of the network is considered.

The equation commonly used to relate conditions between two distinct locations in a straight, non-branching section of piping is Bernoulli's equation. If the fluid contained in the first location is labeled fluid element one, and the fluid contained in the second location is labeled fluid element two, then Bernoulli's equation relates conditions in fluid element one to those in fluid element two. Mott (1979) gives one form of Bernoulli's equation as

$$\frac{p_1}{\gamma} + z_1 + \frac{v_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{v_2^2}{2g} \qquad (2)$$

where p_α is atmospheric pressure [N/m^2], γ is the specific weight of the fluid [N/m^3], z_α is the elevation of the element of fluid [m], v_α is the velocity of each element of fluid [m/s], and g is the acceleration due to gravity [m/s^2].

The system for which equation (2) applies has no lost work. In a cave system, lost work is a significant factor that dramatically affects the energy in a system, so it is necessary to modify equation (2) to introduce lost work into the model. Bernoulli's equation is a conservation of energy equation, and scaling the total energy at one location introduces the effects of lost work and gives the total energy at the second location. Jernigan (1997) derived

$$(1 + \beta \Delta T)\left(\frac{p_1}{\gamma_1} + z_1 + \frac{v_1^2}{2g}\right) = \left(\frac{p_2}{\gamma_2} + z_2 + \frac{v_2^2}{2g}\right) \qquad (3)$$

as a modified version of Bernoulli's equation. In equation (3), β [C^{-1}] is an expansion coefficient dependent upon the substance and ΔT is the temperature differential T_2-T_1 [C]. Equation (3) implies that the work lost in the system is manifested by a temperature difference between locations one and two. Further, γ, the specific weight of the fluid, is temperature dependent and, therefore, different for each of the two elements of fluid. This difference is denoted by the subscripts on γ.

To use equation (3) as a model of airflow in the Historic Section of Mammoth Cave, the airflow must satisfy certain properties.

1. Air flows in the Historic Section as though it were incompressible.
2. With respect to energy exchange, the section of the cave considered is neither a source nor a sink for other sections of the cave.
3. The air in this section of the cave and the cave itself are thermodynamically isolated.
4. There is no energy loss due to friction between molecules within the fluid or between the fluid and its surroundings.

Assumption 1 is satisfied since air pressures and air velocities in Mammoth Cave are small enough in magnitude that there is no compression of the fluid occurring. Assumption 4 holds since the total energy of the system considered is very large, and, while there may be significant energy loss due to friction, it should be small when compared to the total energy of the system. Assumptions 2 and 3 do not hold for the Historic Section of Mammoth Cave. The model developed in this section is modified in the following section to include the effects described in assumptions 2 and 3.

The data from the CAM station in Booths Amphitheater is now predicted using data from Houchins Narrows. There are three temperature probes at the Booths Amphitheater CAM station, and the air temperature data considered are those obtained from the probe near the War of 1812 era saltpeter leaching vats. This probe is ~1 m above the floor and is referred to as the "vat probe."

To predict air temperatures at the vat probe using atmospheric data from Houchins Narrows, it is assumed that the velocity of the airflow at the vat probe is constant with time, so that $v_1 = c_1$, where c_1 is some constant. Further, Jernigan (1997) showed that airflow in Mammoth Cave fluctuates in response to variations in air temperature on the surface and not to changes in barometric pressure. Hence, it is assumed that airflow between two points in the cave system is driven by the temperature differential between those two points. Therefore, by existence of the term ΔT in equation (3), the resultant pressure gradient is indirectly measured, and the erroneous simplifying assumption that $p_1 \approx p_2 = c_2$, for c_2 a constant, is warranted. Last of all, the variation in the value of γ is insignificant over the range of temperature values that are commonly realized in Mammoth Cave, so the value of γ is taken to be constant over space and time. This means $\gamma_1 = \gamma_2 = c_3$ for some constant c_3.

Substitution of these values into equation (3) gives the following

$$(1 + \beta \Delta T)\left(\frac{c_2}{c_3} + z_1 + \frac{c_1^2}{2g}\right) = \left(\frac{c_2}{c_3} + z_2 + \frac{v_2^2}{2g}\right) \tag{4}$$

Since β, z_α, and g are constants, by equation (1), equation (4) reduces to

$$T_B = aF_H^2 + c + T_H \tag{5}$$

where a and c are constants, and T_B and T_H are air temperatures at the vat probe and Houchins Narrows, respectively. Equation (5) is called the Phase 1 Bernoulli model since it is the first step in a sequence of refinements based upon Bernoulli's equation. The quantities that must now be determined are the values of a and c. This may be accomplished through regression analysis of Houchins Narrows and Booths Amphitheater CAM data using the set of basis functions $\{1, F_H^2\}$. Performance of this regression analysis for Julian days 1-20 of 1997 gives the set of results shown in table 1.

Table 1. The phase 1 Bernoulli model applied to Houchins Narrows and Booths Amphitheater data.

Constant	Coefficient	P	R-squared	Normality	Constant Variance	Standard Error of Estimate
a	.010	<.001	.73	Failed	Failed	1.96
c	4.45	<.001				

The value of R-squared equaling only .73, together with the model's failure of the normality and constant variance tests, indicates that this model is not effective in predicting data at the vat probe. Further, the standard error of the estimate is large (1.96°C). This model is inaccurate, as expected, and is modified in the next section to include the effects of other natural phenomena that are influencing the interrelationship of data from Houchins Narrows and those at the vat probe.

A REFINED MODEL OF AIRFLOW IN MAMMOTH CAVE

In this section, the Phase 1 Bernoulli model is modified to include the effects outlined in assumptions 2 and 3. These modifications will improve the values of R-squared realized by the Phase 1 Bernoulli model.

The assumption that no energy is lost to or gained from other passages when air moves from Houchins Narrows to Booths Amphitheater is incorrect since air may flow into or out of Audubon Avenue at the Rotunda. If $\mathbf{F}_H$, $\mathbf{F}_B$, and $\mathbf{F}_A$ represent air flux in Houchins Narrows, air flux near Booths Amphitheater, and air flux in Audubon Avenue, respectively, then the relationship between these variables is $\mathbf{F}_H = \mathbf{F}_A + \mathbf{F}_B$.

Since $\mathbf{F}_B$ is assumed to be approximately zero, then it is true that $\mathbf{F}_A \approx \mathbf{F}_H$. This approximation is transformed into equality by scaling the value of $\mathbf{F}_H$, so that

$$\mathbf{F}_A = (b+m)\mathbf{F}_H = b\mathbf{F}_H + m\mathbf{F}_H\,, \tag{6}$$

where b and m are constants so that b+m is also a constant. Introducing the rightmost representation of $\mathbf{F}_A$ into the Phase 1 Bernoulli model introduces the effects of energy exchange with adjoining passageways.

Further, since airflow in Mammoth Cave is driven by the

chimney effect, temperature differentials between air in the cave and air outside the cave influence airflow patterns throughout the cave. It will be assumed that air flux in Houchins Narrows is directly proportional to the temperature difference between the air in Houchins Narrows and the air outside the cave system. Hence, if T_O represents the temperature of the air outside the cave system and n is a constant, then the following relationship may be established:
$\mathbf{F}_H = n\,(T_H - T_O)$.

Outdoor air temperature at Mammoth Cave National Park is collected on an hourly basis. This time scale does not correspond to the time scale of the CAM station measurements. As a convenience and as an assumption in the current model, the term T_O will be discarded and the above equation reduces to:
$\mathbf{F}_H = n\,T_H$.

Substituting $\mathbf{F}_H$ into the rightmost representation of $\mathbf{F}_A$ in equation (6) yields

$$\mathbf{F}_A = b\mathbf{F}_H + (d-1)T_H\,, \tag{7}$$

where d = mn+1. This relationship replaces equation (6) as source for the basis functions being introduced into the Phase 1 Bernoulli model to account for energy lost or gained due to air exchange with Audubon Avenue.

Assumption 3 is that there is no energy exchange between the air in cave passages and the rock that defines the cave passages. Empirical evidence indicates that this is not the case. The amount of energy exchange between the air in Houchins Narrows and the rock in Houchins Narrows is assumed to be directly proportional to the temperature differential between these substances. Hence, if

$$\Delta T_{H\text{-}R} = T_H - T_R$$

represents the temperature differential obtained by subtracting the temperature of the rock in Houchins Narrows from the temperature of the air in Houchins Narrows, then a constant multiple of this quantity will represent this energy exchange. Further, the rate of energy exchange is assumed constant as air moves from Houchins Narrows to Booths Amphitheater, so that the total energy lost to, or gained from, the surrounding rock is approximately

$$\mathrm{TotE}_{H,R} = k \cdot \mathrm{dist} \cdot \Delta T_{H\text{-}R}$$

where "dist" is the distance from Houchins Narrows to Booths Amphitheater. The constant k is the scaling factor used to define the proportionality relationship between the temperature differential and energy exchange in Houchins Narrows. The term k×dist will be reduced to the constant term g. Hence, the term that describes energy exchange between the air and the rock as the air moves between Houchins Narrows and Booths Amphitheater is given by

$$\mathrm{TotE}_{H,R} = g\Delta T_{H\text{-}R} \tag{8}$$

and this term is introduced into the Phase 1 Bernoulli model.

Adding the new basis functions given by equations (7) and (8) to the existing Phase 1 Bernoulli model, the revised model for predicting data at the vat probe is

$$T_B = a\mathbf{F}_H^2 + b\mathbf{F}_H + c + dT_H + g\Delta T_{H\text{-}R} \tag{9}$$

This model is used in the regression analysis performed in the following sections. It will be referred to as the Phase 4 Bernoulli model since its derivation required three refinements to the Phase 1 Bernoulli model.

REGRESSION ANALYSIS AND THE PREDICTION OF AIR TEMPERATURE IN MAMMOTH CAVE

The Phase 4 Bernoulli model was constructed to predict air temperatures at the vat probe in Booths Amphitheater based upon atmospheric data that describe conditions in Houchins Narrows. However, the Phase 4 Bernoulli model contains constants a, b, c, d, and g that are unknown. These constants are obtained by regression analysis of Cave Atmospheric Monitoring data from Houchins Narrows and Booths Amphitheater. In performing this regression analysis, Cave Atmospheric Monitoring data are partitioned into time intervals ~20 days in length. Some results obtained by using the Phase 4 Bernoulli model along with these CAM data are shown in table 2.

In the table of results, the columns of data detail characteristics of the regression fits. The first column labels the data sets being used in the analysis. For example, the results for the data set "ba(hn)91-110,1996" present the results when Booths Amphitheater vat probe data are predicted using Houchins Narrows data for Julian days 91-110 of 1996. The next five columns of data are the values of the coefficients a, b, c, d, and g obtained through regression analysis. The seventh column contains the value of R-squared when regression analysis is performed on the indicated data set. The eighth and ninth columns describe whether the residuals obtained using the model passed (indicated by a 'P') or failed (indicated by an 'F') the normality and constant variance tests, respectively. The final column gives the standard error produced when the model is applied to the indicated data set.

Figure 2 shows the measured air temperature values at the vat probe for Julian days 1 through 20 of 1997 along with those predicted by the Phase 4 Bernoulli model and the coefficients listed in table 2. The model accurately predicts air temperature at the vat probe.

Values of R-squared are maximized during the winter months; in contrast, winter is also when standard errors are at a maximum. Cave geometry and the fact that airflow in Mammoth Cave is driven by the chimney effect explain this

Table 2. Prediction of air temperature at the vat probe with the phase 4 Bernoulli model.

Data Set	a	b	c	d	g	R-squared	Normality	ConstVar	Std Error
ba(hn)91-110,1996	0.00022	-0.029	4.94	0.46	-0.42	0.96	P	F	0.15
ba(hn)111-130,1996	0.00018	-0.025	4.43	0.54	-0.48	0.91	P	F	0.18
ba(hn)131-150,1996	-0.00032	-0.025	4.24	0.58	-0.58	0.91	P	F	0.15
ba(hn)151-170,1996	-0.00038	-0.026	2.70	0.75	-0.77	0.82	P	F	0.11
ba(hn)195-210,1996	-0.000032	-0.018	7.47	0.28	-0.20	0.49	P	F	0.1
ba(hn)221-240,1996	-0.00022	-0.023	-1.40	1.10	-1.08	0.59	P	F	0.083
ba(hn)245-265,1996	-0.000036	-0.013	5.79	0.45	-0.46	0.83	P	F	0.074
ba(hn)270-290,1996	0.000014	-0.013	6.24	0.41	-0.37	0.92	P	P	0.1
ba(hn)321-346,1996	0.00009	-0.026	6.94	0.31	-0.25	0.95	P	F	0.12
ba(hn)348-366,1996	0.00048	-0.030	6.85	0.32	-0.25	0.98	P	F	0.12
ba(hn)1-20,1997	0.00035	-0.020	6.04	0.42	-0.37	0.99	P	P	0.19
ba(hn)21-40,1997	-0.00011	-0.030	6.34	0.32	-0.26	0.94	P	F	0.14
ba(hn)41-60,1997	-0.00065	-0.018	5.99	0.41	-0.37	0.98	P	F	0.12
ba(hn)61-77,1997	-0.00006	-0.021	6.83	0.27	-0.21	0.89	P	F	0.15

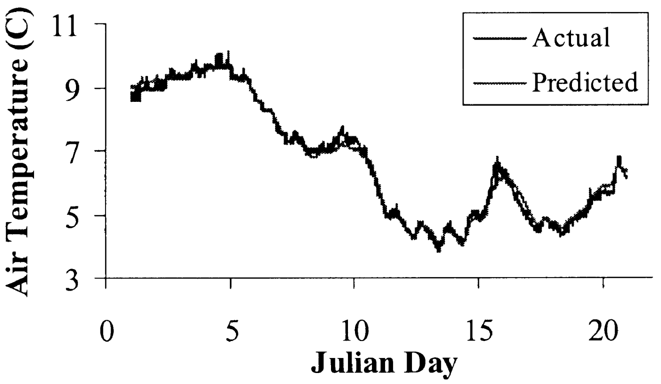

Figure 2. Actual and predicted (using equation (9) and the coefficients in table 2) air temperatures at the vat probe (Julian days 1 through 20, 1997).

anomaly. Since Houchins Narrows is adjacent to the Historic Entrance (a lower entrance into the cave system), air flows from the outdoors into Houchins Narrows in the winter. For the same reason, air flows out of the cave system through this passage during the summer. So, during the winter, fluctuations in the outdoor air temperature are immediately manifested in Houchins Narrows, as they also are in Booths Amphitheater. During the summer, however, the air flowing past these sites has been cooled and thermodynamically stabilized by the cave system. The small fluctuations in air temperature in Booths Amphitheater and Houchins Narrows do not reflect the large fluctuations in air temperature on the surface.

Although it was designed to model air temperatures at the vat probe in Booths Amphitheater, the Phase 4 Bernoulli model can be used to predict air temperatures at other points throughout the Historic Section of Mammoth Cave. Some sites where this is possible are Wrights Rotunda, River Hall, Rafinesque Hall, and the Corkscrew. Each of these sites (with the exception of Rafinesque Hall) has one air temperature probe on site. At Rafinesque Hall, there are two air temperature probes: one at the floor and one at the ceiling of the passage. The results for Rafinesque Hall presented in table 3 are those obtained when using the Phase 4 Bernoulli model to predict air temperature at the ceiling probe.

Some sample results are shown in table 3. The Phase 4 Bernoulli model does well when predicting air temperatures in Rafinesque Hall (abbreviated ra) and the Corkscrew (co), both sites that are close to Houchins Narrows. Also, the prediction of air temperature at the River Hall (ri) CAM station is good, with a value of R-squared equaling 0.84. Wrights Rotunda (wr), however, is a great distance from Houchins Narrows, and this may explain the low value of R-squared (0.32) shown in the table.

SUMMARY AND CLOSING REMARKS

The authors have presented a mathematical model that predicts air temperature in the Historic Section of Mammoth Cave. Based upon basic laws of physics and Cave Atmospheric Monitoring data, this model accurately predicted air temperature at sites near Houchins Narrows during the winter. In the prediction of air temperature at the vat probe in Booths Amphitheater, values of R-squared obtained during the warmer, summer months were not as high as those obtained during the cooler, winter months. In contrast, standard errors are highest when the values of R-squared are highest. Air temperature could also be predicted at sites other than Booths Amphitheater, and the mathematical model that predicts these conditions also resulted in high values of R-squared.

In the development of the Phase 4 Bernoulli model, the effects of outdoor air temperature upon air temperature at the vat probe in Booths Amphitheater were disregarded. However, hourly air temperature measurements are available from a weather station located on the surface and near the boundary of Mammoth Cave National Park. One improvement to the model

Table 3. Prediction of air temperature at sites in the Historic Section of Mammoth Cave.

Data Set	a	b	c	d	g	R-squared	Normality	ConstVar	Std Error
co(hn) 1-20, 1997	-0.000013	0.0026	7.92	0.23	-0.26	0.98	P	P	0.16
ba(hn) 1-20, 1997	0.00035	-0.02	6.04	0.42	-0.37	0.99	P	P	0.19
wr(hn) 1-20, 1997	-0.00032	0.11	8.42	0.34	-0.62	0.32	F	F	2.43
ri(hn) 1-20, 1997	-0.000021	0.0073	7.37	0.077	-0.12	0.84	F	F	0.14
ra(hn) 1-20, 1997	-0.000013	0.0052	5.95	0.18	-0.2	0.96	P	F	0.14

presented is the inclusion of outdoor air temperature as a new basis function. This inclusion should improve the regression results obtained using the Phase 4 Bernoulli model, particularly during the warmer months.

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