THE UNDERSIDE OF RIVER ICE, ST. CROIX RIVER, WISCONSIN

By KEVIN L. CAREY, Madison, Wis.

Abstract.—Ripplelike and dunelike features have been found during a second winter on the underside of the ice cover on the St. Croix River, Wis. Profiles of the features and measurements of ablation and accretion show that the features began to form when the underside of the ice was accreting, and continued their development during ablation. The Belokon-Sabaneev formula, $n_I = (2n^{3/2} - n_B^{3/2})^{2/3}$, satisfactorily shows the relationship between the roughness coefficients for the ice, the bed, and the total channel. Calculations of the roughness coefficient for the underside of the ice, n_I, range from 0.0039 to 0.0142, except for one calculation which indicates no retardation of the flow by the ice cover. This anomaly is believed to be caused by an incorrect estimate of the effective size of the roughness projections of the bed. Caution is therefore given that the computed (published) values of the ice-roughness parameters are slightly smaller than actual.

Relief features that appear analogous to some of the bed forms of alluvial streams have been observed on the underside of river ice in a half-mile reach of the St. Croix River near Danbury, Wis. These observations were reported by Carey (1966), along with the method for calculating the roughness coefficient of the underside of the ice. This report presents the findings of an additional winter's study (1965–1966) which supplement the data already reported for the winter of 1964–65.

The first winter (1964–1965) of the study presented conditions that are normally found at the St. Croix River study site except for slightly low monthly average discharges. The second winter's (1965–66) conditions were unusual in two respects: fairly high flow at greater than average depths throughout the winter, and the presence of frazil (slush) ice in the channel beneath the solid ice cover during the first part of the winter.

OBSERVATIONS OF UNDERSIDE OF ICE, 1965–66 WINTER

The frazil ice that was located immediately below the solid ice cover, was stationary or moving imperceptibly. It varied in thickness in different places from several tenths of a foot to several feet and extended to the streambed in some localities. The amount of frazil ice increased downstream through the half-mile reach in which the river widens and velocities diminish slightly. The proportion of the volume of the channel beneath the ice cover that was occupied by frazil ice decreased with time until midwinter, when some frazil ice had become incorporated into the solid ice cover, and the remainder had melted or had been flushed downstream. The frazil ice and (or) greater effective depths of the river (averaging 3–4 feet), compared to those of the previous winter (when depths averaged 2–2.5 feet), apparently did not favor the formation of relief configurations on the underside of the ice during the early part of the winter.

Observations of the underside of a block of ice were made on seven occasions (table 1), spaced at intervals of 6–12 days, and were made in the same part of the reach where ice blocks had been examined the previous winter. The first two observations were made at times when frazil ice was adhering to the underside of the solid ice cover. The underside of this frazil ice was irregular, but had no discernible pattern. On the occasion of the third observation, frazil ice was not present beneath the solid ice cover in the vicinity of the ice block observations; the underside of the observed ice block was fairly smooth and showed no pattern of subtle irregularities. The four succeeding observations (4–7, table 1) showed significant relief on the underside of the ice.

Ripple configurations found on the underside of the four blocks of ice that showed relief during the second winter (4–7, table 1) were no different in form from those observed the first winter. The configurations resembled the ripple and dune patterns found in the bed forms of an alluvial stream flowing over medium to fine sand. However, the first two of these blocks, Nos. 4 and 5, had configurations that were more

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TABLE 1.—*Winter measurements of stream discharge and channel geometry, computed hydraulic parameters, observations of configuration of the underside of the ice, and amount of frazil ice on the St. Croix River near Danbury, Wis.*

[Subscripts B and I refer to bed-section and ice-cover section]

No.	Date (1966)	Discharge Q (cfs)	Mean velocity V (fps)	Cross-sectional area [1]			Wetted perimeter			Hydraulic radius		
				A (ft²)	A_B (ft²)	A_I (ft²)	P (ft)	P_B (ft)	P_I (ft)	R (ft)	R_B (ft)	R_I (ft)
1	Jan. 13	1,123	1.79	627	434	193	463	230	233	1.35	1.89	0.828
2	24	920	1.66	555	427	128	456	227	229	1.22	1.88	.559
3	Feb. 3	908	1.77	514	[2] 522	--------	451	225	226	1.14	[2] 2.32	--------
4	15	1,402	1.98	707	581	126	453	226	227	1.56	2.57	.555
5	21	1,090	1.84	593	585	35	454	227	227	1.31	2.46	.154
6	Mar. 4	1,230	1.95	631	559	72	454	227	227	1.39	2.46	.317
7	10	1,300	1.86	700	507	193	455	228	227	1.54	2.22	.850

No.	Date (1966)	Energy gradient $S_e \times 10^4$ (ft/ft)	Darcy-Weisbach friction factor			Manning roughness coefficient			Ripples on underside of solid ice		Portion of volume of the reach under solid ice cover that was occupied by frazil ice (percent)
			f	f_B	f_I	n	n_B	n_I	Wavelength (ft)	Amplitude (ft)	
1	Jan. 13	3.79	0.0412	0.0575	0.0252	0.0198	0.0247	0.0142	[3]	[3]	27
2	24	3.27	.0373	.0576	.0171	.0186	.0247	.0110	[3]	[3]	18
3	Feb. 3	2.76	.0260	[2] .0530	--------	.0152	[2] .0245	--------	None	None	13
4	15	3.01	.0308	.0507	.0109	.0175	.0244	.0088	Approx. 0.4	Approx. 0.02	2
5	21	2.76	.0275	.0517	.0032	.0160	.0244	.0039	0.5	0.02–0.03	1
6	Mar. 4	3.09	.0291	.0516	.0066	.0167	.0244	.0062	0.5–0.7	0.06–0.08	0
7	10	3.24	.0372	.0538	.0206	.0192	.0245	.0129	0.6–0.7	0.07–0.09	0

[1] Includes only flow area, not area occupied by frazil ice.
[2] See discussion in text.
[3] Frazil ice present under solid ice cover.

subdued than any examined the previous winter. The wavelength and amplitude of the features increased progressively from about 0.4 foot and about 0.02 foot, respectively, for block 4, to as much as 0.6–0.7 foot and 0.07–0.09 foot, respectively, for the last block, No. 7. As shown in table 1, the discharge and mean cross-sectional area (an approximate indicator of mean depth when divided by the width) were largest for block 4, dropped to low values for block 5, and then increased for blocks 6 and 7. The mean velocity, however, was highest for the first of the four observations of configurated ice (No. 4), dropped for the second, rose for the third, and dropped again for the final observation. In short, then, the progressive increase in the textural roughness of the four configurated blocks is not readily explained by progressively increasing or decreasing velocities or depths, on the basis of this limited data.

Profiles of the features, and data on ablation and accretion of the underside of the ice, discussed later, indicate that during the time interval between the first two block observations (Nos. 4 and 5), accretion of ice occurred. Net ablation of the ice surface took place during the time intervals between the second, third, and fourth of these observations.

Changes in the ice ripples

An attempt was made during the 1965–66 winter season to study more closely the change of the ice ripples with time. Profiles (longitudinal to the stream) of the ripples (transverse to the stream) on the underside of the ice were obtained at various intervals of time at fixed locations on the ice cover. This was done by passing a graduated J-shaped steel bar through a slot cut in the ice by a chainsaw on a line parallel to the flow direction. The bar was turned at right angles to the slot, and pulled up so the tip of the J was in contact with the underside of the ice. The distance from the bottom of the ice to a common datum (that is, the value of the graduation on the J-shaped bar where it intersected the common datum) was noted at a number of equally spaced points along the slot. Thus a profile of the underside of the ice along a line parallel to, and 15 inches away from the slot (15 inches is the width of the J) could be drawn through these points plotted on graph paper. The common datum (with respect to the ice) and horizontal control were maintained by two vertical wood posts frozen in place in the ice about 7 feet apart, across the top of which a graduated rod was placed. The posts extended about 2 feet above the ice top surface. The location of the rod was fixed by a pin in one post and a corresponding hole in the horizontal graduated rod. The slot, cut by the chainsaw each time a profile was obtained, was about 6 feet long and located between the vertical posts; thus it was directly below the horizontal graduated rod.

By this procedure outlined above, the underside of the ice was profiled along the same line each time, and ablation or accretion, as well as any possible migration of the ripples, could be detected.

Because the ripples that first developed were so subtle, accurate profiles were difficult to obtain. By the time that the amplitude of the ripples increased, general ablation of the underside of the ice had begun. An example of a portion of the profiles is shown in figure 1. The available data appear to indicate that during the period of ablation, melting on the upstream faces of the ripples may be greater than on the downstream faces. This lends some support to the hypothesis that during a period of little net ablation or net accretion, the ripples may migrate downstream by means of melting on the upstream faces and freezing on the downstream faces, closely analogous to the migration of sediment ripples and dunes by erosion on the upstream faces and deposition on the downstream faces. Downstream migration could also occur if melting on the upstream faces took place while the downstream faces neither froze nor melted; this would result in general ablation of the underside of the ice.

Net ablation or accretion

In addition to the detailed studies of the ice ripples during the 1965–66 winter season, a record was obtained of the net ablation or accretion at the top and the bottom of the ice at given points, over successive intervals of time. This was done with ablation-accretion (A–A) gages. Each gage was made up of a ¼-inch-diameter wooden dowel 3 feet long and equipped with a 5-inch length of 0.045-inch-diameter stainless steel stranded wire, which pierced the dowel perpendicularly 6 inches from one end. The dowel was graduated in both directions from the crosswire. In use, a vertical hole was drilled through the ice with a carpenter's brace and wood bit with bit extension. Then the dowel was inserted into the hole, with the end having the wire pointing downward. The ends of the flexible wire bent back parallel to the dowel until they passed through to the bottom of the hole, then they sprang out and the wire was horizontal. The dowel was lifted up to a position, determined by feel, where the wire was just touching the bottom of the ice, and the dowel was secured in this position with a clothes pin at the top surface of the ice. The thickness of the ice was noted by the location of the clothes pin on the graduated dowel. After an interval of time during which the A–A gage froze into place (usually the period between discharge measurements), a block containing the A–A gage was cut out. Any net ablation or accretion of the top surface was measured relative to the clothes pin, and after the block was turned over, net ablation or accretion of the underside was measured relative to the wire. At this time, another A–A gage was installed in previously undisturbed ice near the first gage, to be cut out at the end of the next interval of time.

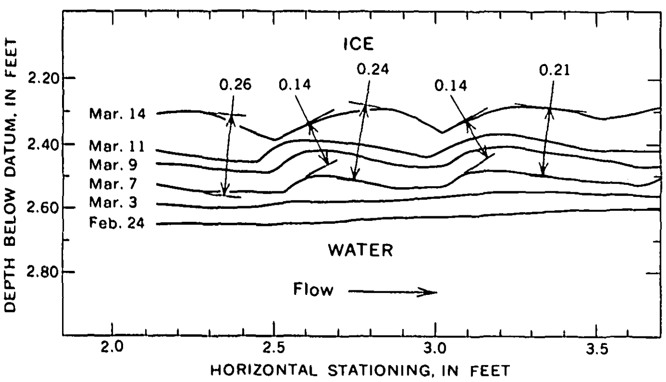

FIGURE 1.—Example of a portion of ice-ripple profiles obtained by the method described in text. Dimensions, in feet, are amount of ablation (Feb.–Mar., 1966) measured approximately perpendicular to the ice surface at points on the upstream and downstream faces of the ripples.

Minor accretion occurred at the top of the ice during two of the six intervals in the 1965–66 season. This was the result of the snow cover becoming saturated in the bottom portion, and then freezing. Accretion at the bottom was a total of 0.9 foot from early January to early February. Little change occurred at the bottom through late February, and net ablation from then until early March amounted to 0.4 foot.

CALCULATIONS OF ROUGHNESS, 1965–66 WINTER

Roughness coefficients and friction factors for the underside of the ice during the 1965–66 winter were computed by a method previously reported (Carey, 1966). In 1964–65, values of the Manning roughness coefficient for the underside of the ice, n_I, ranged from 0.0100 to 0.0281, being generally lower in the early part of the period of ice cover. That winter the value of the Manning roughness coefficient for the streambed, n_B, was nearly constant at 0.0251. For 1965–66, the values of n_I in 6 of 7 discharge measurements ranged from 0.0039 to 0.0142; n_B was again nearly constant, but had values averaging 0.0245. The computation of n_I for 1 (Feb. 3, 1966) of the 7 measurements gave unrealistic results (discussed later), and implied that n_I was in a sense negative. In this second winter, n_I was not at all times a meaningful parameter of ice roughness, being more a coefficient for the combined effect of the solid ice undersurface and the variable amounts of frazil, when the latter was present.

A formula showing the relationship between bed roughness, ice roughness, and the total, composite, or combined roughness would be useful in determining the Manning roughness coefficient for the underside of river ice, in that it would eliminate the need for the computational method previously developed (Carey, 1966). Nezhikhovskiy (1964), in a review of this topic, presented three such empirical formulas. One is a general average formula:

$$n=\frac{n_B+n_I}{2} \text{ or } n_I=2n-n_B,$$

another is known as the Pavlovskiy formula:

$$n_I=\sqrt{(2n^2-n_B^2)},$$

and the third is the Belokon-Sabaneev formula:

$$n_I=(2n^{3/2}-n_B^{3/2})^{2/3}.$$

The data from both years of this study are described well by the Belokon-Sabaneev formula. For 20 discharge measurements and the corresponding roughness calculations, the values of n_I computed by this formula differ by no more than ±2.6 percent from the values determined by the method of the writer. The average absolute error is only 0.6 percent. Thus the use of the Belokon-Sabaneev formula appears entirely justified, if a satisfactory mean value of n_B is known.

DISCUSSION

In the previously developed computational method (Carey, 1966) the mean cross section of the reach is assumed to be divided into two parts: one section (the bed section) exerting shear on the streambed, and the other section (the ice-cover section) exerting shear on the underside of the ice. The analysis of the anomalous data from the discharge measurement of February 3, 1966 indicated that A_B, the area of the bed section, was larger than A, the area of the entire cross section. Since the geometry of the situation requires that $A_B+A_I=A$ (A_I being the area of the ice-cover section), a negative value of A_I was indicated. If A_I were zero, it would mean the ice cover was offering no resistance to flow, and so a negative value of A_I would lead to the absurd conclusion that the ice cover was actually spurring the flow rather than retarding it. This would imply that n_I was in a sense negative.

The explanation for the above anomaly can be found in the method by which the values of n_I are computed. A full development of this method, which is outlined below, is given in the author's previous paper (1966).

It is assumed that the Kármán-Prandtl resistance equation for turbulent flow in rough pipes, a relationship between the Darcy-Weisbach friction factor and the relative roughness of a conduit, can be applied to the flow in the bed section of the ice-covered stream. This equation takes the form

$$\frac{V_B}{\sqrt{8gR_BS_e}}=2\log_{10}\left(\frac{2R_B}{k_B}\right)+1.74,$$

in which V_B is the mean velocity in the bed section (assumed equal to the mean velocity in the entire cross section), g is the acceleration of gravity, R_B is the hydraulic radius of the bed section, S_e is the energy gradient of the flow in the reach, and k_B is the effective size of the roughness projections of the bed. The value of k_B is determined from discharge measurements made during open-water periods and is assumed constant throughout the winter. Thus for each discharge measurement made during ice-cover periods, all the terms of the above equation are known except R_B, which can be solved for by trial and error. With R_B known for a particular discharge measurement, A_B is found by the common relation $A_B=R_BP_B$, where P_B is the wetted perimeter of the bed section, a quantity determined by the field data for the discharge measurement. Then, as indicated earlier, A_I is found by the requirement that $A_B+A_I=A$. With A_I known, R_I (the hydraulic radius of the ice-cover section) follows from the common relation $R_I=A_I/P_I$, where P_I is the wetted perimeter in the ice-cover section. With R_I evaluated, n_I follows from the Manning equation.

The values of n_I depend entirely upon an accurate estimate of k_B. In this and the previous report (Carey, 1966), a value of $k_B=0.23$ foot was used; as mentioned in the previous report the ice-roughness parameters must be termed provisional, inasmuch as k_B was estimated from limited data.

In the data of February 3, 1966, A_B exceeded A because R_B was too large. A smaller value of k_B would produce a smaller value of R_B, so it is evident that the estimate $k_B=0.23$ foot is too large. A value at least as small as 0.20 foot would be needed to produce a positive value of A_I for the discharge-measurement data in question.

On the assumption that k_B is constant for the streambed of the St. Croix River in the study reach, and that its correct value is as small or smaller than 0.20 foot, the values of n_I and the Darcy-Weisbach friction factor for the ice, as published in this and the previous reports, are slightly lower than the correct values. Correspondingly, the values of the bed roughness parameters are too high. The published ice-roughness parameters are nonetheless useful for comparing one observation with another, even though they are slightly inaccurate in an absolute sense.

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